

PHYS 798C Spring 2022

Lecture 26 Summary

Prof. Steven Anlage

I. FLUCTUATION CONDUCTIVITY ABOVE T_c

The resistance vs. temperature curves show precursor effects as the transition temperature is approached from above. The resistance is observed to drop at temperatures above T_c , to a degree that depends on the dimensionality of the material (0D, 1D, 2D, 3D). We treat the conductivity above T_c as a linear combination of mean-field conductivity (entirely due to normal state physics) and fluctuation conductivity (due to superconducting fluctuations):

$$\sigma = \sigma_{MF} + \sigma^{fluc}.$$

The idea is that the material borrows $k_B T$ of energy from the thermal bath temporarily and creates a superconducting fluctuation of limited size and limited lifetime somewhere in the material. While this fluctuation exists it creates a small zero-resistance pathway for current, leading to a drop in global resistance of the sample. These fluctuations occur rapidly and in many locations around the sample, leading to clearly measurable effects when measured on the time scales of seconds.

We will work with the Fourier transform of $\langle |\psi|^2 \rangle$. Consider the GL free energy expansion above T_c ($\alpha > 0$), where the fluctuating order parameter is expected to be small. In this case we can ignore the $|\psi|^4$ term, and we will take the vector potential and external fields to be zero. This leaves a free energy expansion of,

$$f = \alpha |\psi|^2 + \frac{\hbar^2}{2m^*} (\nabla \psi)^2.$$

Now take the Fourier transform of ψ ,

$$\psi(\vec{r}) = \sum_k \psi_k e^{i\vec{k}\cdot\vec{r}}. \text{ This gives for the free energy difference of the sample,}$$

$$F = \int f dV = \sum_k \left\{ \alpha + \frac{\hbar^2 k^2}{2m^*} \right\} |\psi_k|^2.$$

Now assume equipartition of energy to each degree-of-freedom of the fluctuating order parameter. This means that every term in the sum on k will have on average $k_B T$ of energy, or in other words,

$$\langle |\psi_k|^2 \rangle = \frac{k_B T / \alpha}{1 + k^2 \xi_{GL}^2} \frac{1}{V}, \text{ where we have used the fact that } \xi_{GL}^2 = \frac{\hbar^2}{2m^* \alpha}.$$

The k -dependence suppresses the short wavelength fluctuations. In fact we should introduce a cutoff in the k -sum because wavenumbers larger than $1/\xi_{GL}$ cannot be expected to contribute.

How big are the areas that fluctuate in to the superconducting state above T_c ? Below T_c the order parameter is assumed to be long-range phase coherent. Above T_c one can calculate the two-point correlation function $g(\vec{r}, \vec{r}') \equiv \langle \psi^*(\vec{r}) \psi(\vec{r}') \rangle$, which is found to be $g(\vec{r}, \vec{r}') = \frac{m^* k_B T}{2\pi \hbar^2} \frac{e^{-R/\xi_{GL}(T)}}{R}$, where $R = |\vec{r} - \vec{r}'|$ (see Tinkham p. 300). The typical size of the fluctuation is on the order of the GL coherence length above T_c , with $\xi_{GL}^2 = \frac{\hbar^2}{2m^* \alpha}$ and $\alpha > 0$.

II. BRIEF INTRODUCTION TO TIME-DEPENDENT GINZBURG-LANDAU THEORY (TDGL)

Above T_c the equilibrium value of the order parameter is zero, $\psi_0 = 0$. Any fluctuation results in a non-zero order parameter, as we assume $\psi = \psi_0 + \delta\psi$. TDGL says that such a fluctuation will relax back to zero exponentially in time: $\frac{d\psi}{dt} = -\frac{1}{\tau_0} \psi$. The result for the $k = 0$ momentum relaxation time is

$$\tau_0 = \frac{\pi \hbar}{8k_B(T-T_c)} = \frac{3ps-K}{(T-T_c)}. \text{ (This result is derived for gapless superconductors by M. Cyrot, Rep. Prog. Phys. 36, 103 (1973).)}$$

Now consider the GL equation corresponding to the free energy expansion given above,

$$\alpha \psi - \frac{\hbar^2}{2m^*} \nabla^2 \psi = 0, \text{ or after dividing through by } \alpha,$$

$$\xi_{GL}^2 \nabla^2 \psi - \psi = 0. \text{ The TDGL generalization of this equation is,}$$

$$\frac{d\psi}{dt} = -\frac{1}{\tau_0} (1 - \xi_{GL}^2 \nabla^2) \psi.$$

Generalize the Fourier transform of the order parameter to a time-dependent version,

$\psi(\vec{r}, t) = \sum_k \psi_k e^{i\vec{k}\cdot\vec{r}} e^{-t/\tau_k}$. Putting this into the generalized TDGL equation yields, $\tau_k = \frac{\tau_0}{1+k^2\xi_{GL}^2}$. This shows that short wavelength fluctuations will decay more quickly.

III. FLUCTUATION CONDUCTIVITY IN TDGL

Tinkham derives the fluctuation conductivity above T_c using the Kubo formalism, which examines the current-current correlation function vs. distance. Basically, high correlations mean high conductivity. Here we use a simple TDGL argument. In analogy with the Drude expression for the mean field conductivity, $\sigma_n = \frac{ne^2\tau}{m}$, let's try,

$\sigma^{fluc} = \frac{(e^*)^2}{\hbar^2} \sum_k^{k_{cutoff}} \langle |\psi_k|^2 \rangle \frac{\tau_k}{2}$, where the factor of 2 comes from the fact that ψ_k^2 decays twice as fast as ψ_k .

Substituting in the results for $\langle |\psi_k|^2 \rangle$ and τ_k from above yields,

$\sigma^{fluc} = \frac{(e^*)^2}{\hbar^2} \frac{\xi_{GL}^2}{V} k_B T \tau_0 \sum_k^{k_{cutoff}} \frac{1}{(1+\xi_{GL}^2 k^2)^2}$. The outcome of the sum (turned in to an integral) depends on the dimensionality d of the system.

The results are,

$$d = 3: \sigma_{3D}^{fluc} = \frac{e^2}{\hbar\xi_{GL}(0)} \left(\frac{T}{T-T_c}\right)^{1/2} \times \begin{cases} 1/4 & \text{This approximation} \\ 1/32 & \text{Kubo} \end{cases}$$

where we write $\xi_{GL}(T) = \xi_{GL}(0) \sqrt{\frac{T}{T-T_c}}$.

$$d = 2: \sigma_{2D}^{fluc} = \frac{e^2}{\hbar t} \left(\frac{T}{T-T_c}\right) \times \begin{cases} 1/8 & \text{This approximation} \\ 1/16 & \text{Kubo} \end{cases}$$

where t is the film thickness. Note that the fluctuation conductance $\sigma_{2D}^{fluc} t$ is universal, meaning that it is independent of all sample details when plotted vs. $\epsilon = (T - T_c)/T_c$.

$$d = 1: \sigma_{1D}^{fluc} = \frac{e^2}{\hbar} \left(\frac{T}{T-T_c}\right)^{3/2} \frac{\xi_{GL}(0)}{A_c} \times \begin{cases} 1/8 & \text{This approximation} \\ \pi/16 & \text{Kubo} \end{cases}$$

where A_c is the cross sectional area of the 1D wire.

Note that in all cases the scale of the fluctuation conductivity is set by the quantum of conductance $\frac{e^2}{\hbar} = 243.3\mu S$. Alternatively one can write $\frac{\hbar}{e^2} = 4.11k\Omega$.

The dc fluctuation conductivity diverges at T_c more strongly in lower dimensionality as

$$\sigma_{dD}^{fluc} \propto \frac{1}{(T-T_c)^{(4-d)/2}}.$$

Note that there is no divergence of the fluctuation conductivity in $d = 4$, meaning that mean-field behavior is recovered in four dimensions.

The class web site shows fluctuation conductivity data from 1 and 2-dimensional materials.

IV. THE KOSTERLITZ-THOULESS (KT) PHASE TRANSITION FOR 2D SUPERCONDUCTORS

One does not have true long-range order (LRO) in 1 or 2 dimensions. The 3D BCS ground state is a coherent state of Cooper pairs that maintains phase coherence over effectively infinite distance. In lower dimensions, the phase coherence is less strong. This is quantified by the two-point correlation function for the GL order parameter. In 3D one has $\langle \psi^*(\vec{r})\psi(\vec{r}') \rangle \sim \text{constant}$ as $|\vec{R}| = |\vec{r} - \vec{r}'| \rightarrow \infty$. Above T_c one finds,

$$\langle \psi^*(\vec{r})\psi(\vec{r}') \rangle \sim e^{-R/\xi_N}.$$

In 2D at low temperature one finds, $\langle \psi^*(\vec{r})\psi(\vec{r}') \rangle \sim \frac{1}{R^{\eta(\overline{T})}}$. This difference between the exponential decay above T_c to a power-law decay at low temperatures implies a phase transition. This is the famous ‘‘KT’’ transition that is seen in 2D Coulomb gases and in vortices in thin films of superfluid 4He . The original reference is J. Phys. C 6, 181 (1973).

The theory examines the lowest energy excitations out of the ground state. Individual vortices have an energy that scales logarithmically with the size of the sample. Bound Vortex/Anti-vortex (V/AV) pairs have a much smaller energy, hence they dominate the low temperature properties. As temperature increases, there is an entropic advantage to de-pairing the V/AV pairs and creating un-bound free vortices. This precipitates the KT transition at T_{KT} . Below T_{KT} there is zero resistance in the limit of

current going to zero. In other words the critical current is zero! Above T_{KT} there is finite resistance even in the limit as the current goes to zero.